On the Relativistic Description of the Nucleus

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Abstract

We discuss a relativistic theory of the atomic nuclei in the framework of the hamiltonian formalism and of the mesonic model of the nucleus. Attention is paid to the translational invariance of the theory. Our approach is centered on the concept of spectral amplitude, a function in the Dirac spinor space. We derive a Lorentz covariant equation for the latter, which requires as an input the baryon self-energy. For this we either postulate the most general Lorentz-Poincaré invariant expression or perform a calculation via a Bethe-Salpeter equation starting from a nucleon-nucleus interaction. We discuss the features of the nuclear spectrum obtained in the first instance. Finally the general constraints the self-energy should satisfy because of analyticity and Poincaré covariance are discussed.

1 Introduction

The purpose of this paper, which extends and deepens a previous one[1], is to outline a theory of finite nuclei cast in a Lorentz and translationally invariant form.

Extensive work has been carried out in the past to describe a nucleus in a relativistic regime (see for instance [2, 3] for comprehensive reviews). This has been done essentially in the framework of the Dirac's theory, either phenomenologically[4, 5, 6], i.e. by suitably parametrizing the self-energy felt by a relativistic nucleon when interacting with a nucleus, or microscopically, i.e. by evaluating the self-energy starting from a mesonic theory, like the Bonn potential [7, 8]).

However, the Dirac's theory, as it is usually applied, breaks translational invariance. This appears to be serious at high energies as already indicated by nonrelativistic calculations, which point to the growing importance of the center-of-mass component in the nuclear wave function when the momentum transfer becomes large[9].

A notable extension of the Dirac's theory is represented by the Quantum Hadrodynamics (QHD) of Walecka[10, 11], a quantum field theory model based on a lagrangian which describes nucleons interacting via the exchange of isoscalar scalar (σ) and vector (ω) meson fields.

Notably, QHD has been remarkably successful in dealing both with nuclear matter and finite nuclei[12] in the so-called mean field approximation (MFA), where one replaces the meson field operators by their ground state expectation values ignoring furthermore the negative energy states. However when one goes beyond a MFA to include correlations among nucleons (via, e.g., a Bethe-Salpeter-type equation or truly quantum field theory effects, as those stemming from vacuum polarization) then QHD faces severe problems. Indeed, although great care is paid in its framework to the problem of renormalization, the loop expansion appears to be, to say the least, quite poorly convergent. For example the two loop contribution to the binding energy turns out to be very large at the nuclear matter density[13] because of the unsatisfactory ultraviolet behaviour of the theory, a well known feature common to all the non asymptotically free model. One can of course deal with these difficulties by introducing *ad hoc* form factors to cut off the large momenta entering the diagrams needed to express the physical observables of concern, but in so doing one spoils QHD of its character of a truly hadronic

field theory.

Our approach differs from QHD in several respects. First it is centered on the concept of single particle orbital. In fact, notwithstanding the difficulty of defining a wave function, in particular a single particle one, in a relativistic framework, yet it is still advantageous to deal with the nucleus in terms of a relativistic extension of the quasiparticles, the building blocks of atomic nuclei, as introduced by Landau and then by Migdal[14].

To achieve this goal it is crucial to introduce the so called spectral amplitude ϕ_l, to be later defined, a suitable tool for expressing the nuclear binding energy, the nucleon momentum distribution and therefore the cross sections for a variety of nuclear reactions (for example the exclusive $(e, e'p)$ one[15]) in a Poincaré invariant form.

A second difference from QHD arises from the recognition that the connections between ϕ_l and the observables above referred to are more naturally established in a *hamiltonian* framework where they can be cast in the form of *exact* relations. Of course the problem of the infinities arising from the presence of the Dirac's sea exists and we deal with it empirically by means of *ad hoc* subtractions.

Moreover the hamiltonian scheme has the advantage of allowing us to fulfill more easily (at least formally) Poincaré invariance when treating finite nuclei. As a notable consequence the Dyson's equation for the fermion propagator, from which an equation for ϕ_l is derived, exhibits an algebraic, rather than an integral, structure even for finite nuclei. Obviously here the difficulty is met of properly defining and calculating the self-energy of a baryon or of a meson (see sect. 4).

In this connection we adopt a pragmatic view, namely we write the most general Poincaré invariant self-energy in terms of functions meant to be fixed by the phenomenology[4, 16, 5, 6] and we explore the consequences it entails on the spectrum of the system.

We also investigate the alternative option of *calculating* the self-energy: here our approach is based on a Bethe-Salpeter-type equation[3], with a phenomenological nucleon-nucleus interaction as an input. We thus account, respecting covariance, for the repeated interactions a particle undergoes in its way-out from the nucleus (final state interaction). It is worth noticing that this approach has the merit of transparently displaying for a convenient choice of the interaction and for an infinite homogeneous system how our hamiltonian theory reduces to QHD at the mean field level.

This paper is organized as follows: in sect. 2 we outline our hamiltonian framework, introduce the spectral amplitude and establish the connection of the latter with various physical observables. In sect. 3 we discuss a simple example to illustrate how mesonic degrees of freedom can be accounted for by means of a canonical transformation. In sect. 4 we study the algebraic Poincaré-covariant Dyson's equation for the fermion propagator in a finite system. The Lehmann representation for the propagator is also analyzed. We study then the general structure of the self-energy, its link with the final state interaction via a Bethe-Salpeter equation and its analytic properties. Finally we shortly analyze the resulting energy spectrum of the nuclear system.

2 The formalism

Let us consider a system of nucleons interacting via the exchange of mesons. For simplicity we confine ourselves in this section to consider the exchange of a scalar meson. Euristically the latter simulates the exchange of a pair of pions, a microscopic process viewed as the main source of the nuclear binding. Clearly our model should then be generalized to more realistic situations encompassing several different mesonic fields to account, e.g., for the short range repulsion among nucleons.

Its hamiltonian reads

$$H = H_N^0 + H_\sigma^0 + H' \tag{1}$$

where

$$\begin{aligned} H_N^0 &= \int d^3y\, \overline{\psi}(\mathbf{y})(-i\boldsymbol{\gamma}\cdot\nabla + m)\psi(\mathbf{y}) && (2)\\ H_\sigma^0 &= \frac{1}{2}\int d^3y \left[\dot{\sigma}^2(\mathbf{y}) + (\nabla\sigma(\mathbf{y}))^2 + m_\sigma^2\sigma^2(\mathbf{y})\right] && (3)\\ H' &= g\int d^3y\, \overline{\psi}(\mathbf{y})\sigma(\mathbf{y})\psi(\mathbf{y})\,, && (4) \end{aligned}$$

ψ and σ being the fermion and meson field.

A flaw of (1) stems from the absence of a self-interacting term $\lambda\sigma^4$, needed to bound from below the spectrum of (1). However since, as we shall see in the following, the stability of the σ field is here ensured by the presence of the nuclear medium, the renormalized value of λ can be assumed to be

positive and small: hence the $\lambda\sigma^4$ term is not expected to significantly affect the predictions of the theory and can thus be safely omitted.

A further shortcoming of (1) is connected with the difficulty of defining a vacuum for a bound, finite, relativistic Fermi system. Indeed the standard expansion for a fermion field, namely

$$\psi(\mathbf{y}) = \sum_s \int \frac{d^3k}{(2\pi)^3} \frac{m}{\epsilon_k} \left[u_{ks} a_{ks} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} + v_{ks} b^\dagger_{ks} e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \right] \tag{5}$$

(a_{ks} and b_{ks} being the free nucleon and antinucleon annihilation operators and $\epsilon_k = \sqrt{\mathbf{k}^2 + m^2}$), is not appropriate, being incompatible with the spatial confinement of the atomic nucleus.

However, since we do not pretend to obtain here a solution of the problem of confined states in QFT (in fact we rather search for relations between physical observables and the spectral amplitude) we shall still quantize the fermion field according to (5), but without introducing normal products at all. We shall instead resort to the introduction of compensating terms, in the form of vacuum subtractions, in order to avoid the occurrence of infinities.

On the other hand, as far as the mesons are concerned, we assume their coupling with the fermion field to be so weak to prevent the occurrence of bound meson states. Therefore the expansion

$$\sigma(\mathbf{y}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[c_k e^{i\boldsymbol{k}\cdot\boldsymbol{y}} + c^\dagger_k e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \right] \tag{6}$$

(c_k and $c^\dagger_k$ being the free meson annihilation and creation operators and $\omega_k = \sqrt{\mathbf{k}^2 + m_\sigma^2}$) is likely to be still warranted.

In line with the above considerations we shall rewrite equations (3) and (4) as follows:

$$\begin{aligned} H^0_\sigma &= \frac{1}{2} \int d^3y : \left[\dot\sigma^2(\mathbf{y}) + (\nabla\sigma(\mathbf{y}))^2 + m_\sigma^2 \sigma^2(\mathbf{y}) \right] : \\ &= \int \frac{d^3k}{2\omega_k (2\pi)^3} \, \omega_k c^\dagger_k c_k \qquad (7) \\ H' &= \int \frac{d^3k}{2\omega_k (2\pi)^3} \left(c^\dagger_k \rho_k + \rho^\dagger_k c_k \right) \qquad (8) \end{aligned}$$

where

$$\rho_k = g \int d^3y \, \overline{\psi}(\mathbf{y}) \psi(\mathbf{y}) e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \, . \tag{9}$$

Our choice of the normalization of the fields is such that the commutator

$$[c_k, c^\dagger_{k'}] = (2\pi)^3 2\omega_k \delta(\mathbf{k}-\mathbf{k}') \ , \tag{10}$$

holds for the boson and the anticommutator

$$\left\{a^\dagger_{ks}, a_{k's'}\right\} = (2\pi)^3 \frac{\epsilon_k}{m} \delta_{ss'} \delta(\mathbf{k}-\mathbf{k}') \tag{11}$$

for the fermion fields.

Let us now introduce the nuclear states $|\mathbf{p}, (A)_m>$, identified by the total 3-momentum $\mathbf{p}$, baryonic number A and by a set of internal quantum numbers denoted by m. With the notation $|\mathbf{p}, A>$ we refer to the internal ground state. Since the relativistic normalization of a state is statistic-dependent, we have to choose the parity of A. Let us assume A to be even. Then the normalization reads

$$<\mathbf{p}, A|\mathbf{p}', A>= (2\pi)^3 2p_0 \delta(\mathbf{p}-\mathbf{p}') \ . \tag{12}$$

Of course for the ground state

$$P_\mu|\mathbf{p}, A>= p_\mu|\mathbf{p}, A> \tag{13}$$

and

$$p_0 = \frac{<\mathbf{p}, A|H|\mathbf{p}, A>}{<\mathbf{p}, A|\mathbf{p}, A>} - <0|H|0>= \sqrt{M_A^2+\mathbf{p}^2} \ . \tag{14}$$

Clearly $<\mathbf{p}, A|H|\mathbf{p}, A>$ diverges owing to the contribution to the energy arising from the Dirac sea, hence the vacuum subtraction in (14).

In parallel to (13) for a $(A-1)$ system, left in a quantum state identified by the index l, we have

$$P_\mu|\mathbf{p}, (A-1)_l>= p^l_\mu|\mathbf{p}, (A-1)_l> \ \ , \tag{15}$$

where

$$p^l_\mu \equiv (p^l_0, \mathbf{p}) \tag{16}$$

and

$$p^l_0 \equiv \epsilon^{A-1}_l(\mathbf{p}) = \sqrt{(M^{A-1}_l)^2+\mathbf{p}^2} \ , \tag{17}$$

but now the fermionic normalization

$$<\mathbf{p}, (A-1)_l|\mathbf{p}', (A-1)_{l'}>= (2\pi)^3 \frac{\epsilon^{A-1}_l(\mathbf{p})}{M^{A-1}_l} \delta_{ll'} \delta(\mathbf{p}-\mathbf{p}') \tag{18}$$

holds.

Let us now introduce in the Dirac spinor space the function

$$\phi_{l\boldsymbol{kp}}(y) = C_l(\mathbf{k}, \mathbf{p}) < \mathbf{p} - \mathbf{k}, (A-1)_l|\psi(y)|\mathbf{p}, A > \tag{19}$$

with

$$C_l(\mathbf{k}, \mathbf{p}) = \sqrt{\frac{M_l^{A-1}}{2p_0 \epsilon_l^{A-1}(\mathbf{p} - \mathbf{k})}} \, , \tag{20}$$

the field $\psi(y)$ being now in Heisenberg picture. The above is the key quantity of our formalism, in terms of which we shall express the ground state energy and, in a parallel research, the structure functions of the nucleus. In sect. 4 we shall derive the equation obeyed by ϕ_l.

The amplitude $\phi_{l\boldsymbol{kp}}(y)$ is the tool we need to generalize the quasiparticle concept to a relativistic regime: in fact it can be shown that in the nonrelativistic limit and in the single-particle approximation ϕ_l exactly reduces to a one particle wave function. Furthermore the coefficient $C_l(\mathbf{k}, \mathbf{p})$ is such to account for the difference between the relativistic and nonrelativistic normalization of the states.

Of course the spatial dependence of $\phi_{l\boldsymbol{kp}}(y)$ is fixed by translation invariance. Indeed, from

$$\psi(y) = e^{iP\cdot y}\psi(0)e^{-iP\cdot y} \tag{21}$$

one gets

$$\phi_{l\boldsymbol{kp}}(y) = \phi_l(\mathbf{k}, \mathbf{p})e^{-ik^l\cdot y} \tag{22}$$

where

$$\phi_l(\mathbf{k}, \mathbf{p}) = C_l(\mathbf{k}, \mathbf{p}) < \mathbf{p} - \mathbf{k}, (A-1)_l|\psi(0)|\mathbf{p}, A > \tag{23}$$

and

$$\begin{aligned} k^l &\equiv (\varepsilon_l(\mathbf{k}, \mathbf{p}), \mathbf{k}) & (24) \\ \varepsilon_l(\mathbf{k}, \mathbf{p}) &= p_0 - \epsilon_l^{A-1}(\mathbf{p} - \mathbf{k}) \, . & (25) \end{aligned}$$

Owing to the definition (19) $\phi_l(\mathbf{k}, \mathbf{p})$ transforms under the Lorentz group as a Dirac spinor. Moreover, it can only depend upon the Lorentz-invariant quantities p^2, $(k^l)^2$ and $p \cdot k^l$, but the mass shell conditions (14) and (25) leave only the 3-vectors $\mathbf{p}$ and $\mathbf{k}$ as independent variables.

The quantity $|\phi_l(\mathbf{k},\mathbf{p})|^2$ is the probability density (homogeneous in space and time) of destroying a nucleon (or creating an antinucleon) with 4-momentum k_l in the nuclear ground state leaving the residual nucleus in the quantum state $|\mathbf{p}-\mathbf{k},(A-1)_l>$. Notice also that eqs. (22) to (25) hold valid in the nonrelativistic limit, where the center-of-mass can be correctly defined and its motion accounted for. If however the center-of-mass is not properly singled out, as is the case, e.g., of the nuclear shell model, then obviously the invariance for space translations is lost.

Let us now illustrate how the energy of the nucleus and the momentum distribution of its constituents can be expressed in terms of $\phi_l(\mathbf{k},\mathbf{p})$.

To start with, notice that

$$\sum_l \int \frac{d^3k}{(2\pi)^3}\,|\phi_l(\mathbf{k},\mathbf{p})|^2 = \tag{26}$$
$$= \frac{1}{<\mathbf{p},A|\mathbf{p},A>}\int d^3y\ <\mathbf{p},A|\psi^\dagger(\mathbf{y})\psi(\mathbf{y})|\mathbf{p},A> = A + 4\Omega\int\frac{d^3k}{(2\pi)^3}$$

where the term proportional to the normalization volume Ω arises from the Dirac sea and would be absent had we employed a suitably defined normal product.

Furthermore, from (25) and using closure, one gets:

$$\sum_l \int \frac{d^3k}{(2\pi)^3}\,\varepsilon_l(\mathbf{k},\mathbf{p})\,|\phi_l(\mathbf{k},\mathbf{p})|^2 \tag{27}$$
$$= \frac{1}{<\mathbf{p},A|\mathbf{p},A>}\int d^3y\ <\mathbf{p},A|\psi^\dagger(\mathbf{y})\,(p_0-H)\,\psi(\mathbf{y})|\mathbf{p},A>$$

which, exploiting the commutation relations for the field ψ, can be recast in the form

$$\sum_l \int \frac{d^3k}{(2\pi)^3}\,\varepsilon_l(\mathbf{k},\mathbf{p})\,|\phi_l(\mathbf{k},\mathbf{p})|^2 \tag{28}$$
$$= \frac{1}{<\mathbf{p},A|\mathbf{p},A>}<\mathbf{p},A|\left(H_N^0+H'\right)|\mathbf{p},A>\ .$$

The above relation expresses the energy of the system associated with the fermionic degrees of freedom. This quantity can of course be experimentally inferred from the nuclear separation energy. The "total" energy of the

nucleus, which is also experimentally known, reads instead

$$p_0 = \sum_l \int \frac{d^3k}{(2\pi)^3}\, \varepsilon_l(\mathbf{k},\mathbf{p})\, |\phi_l(\mathbf{k},\mathbf{p})|^2 \tag{29}$$
$$+ \int \frac{d^3k}{2\omega_k(2\pi)^3} \frac{<\mathbf{p},A|\omega_k c_k^\dagger c_k|\mathbf{p},A>}{<\mathbf{p},A|\mathbf{p},A>} - <0|H|0> \quad .$$

One thus sees that in principle the possibility is offered, within our hamiltonian model, to estimate the contribution to p_0 arising from the mesonic components in the nuclear ground state. This is expected to be small: should not this be the case then the mesonic field expansion (6) would not be compatible with the normal product introduced in (7).

Let us now somewhat generalize equation (29). For this purpose we consider again the baryon contribution to the total energy of the nucleus, introducing the analogous of the spectral function in the nonrelativistic theory, namely

$$S(k,\mathbf{p}) = 2\pi \sum_l |\phi_l(\mathbf{k},\mathbf{p})|^2\, \delta(k_0 - \varepsilon_l(\mathbf{k},\mathbf{p})) - S_{\rm vac}(k) \tag{30}$$

which, as it is well known, is connected to the correlation function via a Fourier transform

$$S(k,\mathbf{p}) = \frac{1}{2p_0} {\rm Tr} \int d^4y \ <\mathbf{p},A|\psi^\dagger(y)\psi(0)|\mathbf{p},A>\, e^{-ik\cdot y} - S_{\rm vac}(k) \tag{31}$$

and is crucial for expressing the $(e,e'p)$ phenomenology[15].

In term of the above, and reminding the need of subtracting the vacuum contribution

$$S_{\rm vac}(k) = 8\pi\Omega\delta(k_0+\epsilon_k) \quad , \tag{32}$$

the baryonic part of the energy can then be expressed as follows:

$$p_0^{\rm nucl} = \int \frac{d^4k}{(2\pi)^4} k_0 S(k,\mathbf{p}) \quad . \tag{33}$$

The mesons contribution can be similarly handled by setting

$$S_\sigma^0(k,\mathbf{p}) = 2\pi \frac{<\mathbf{p},A|c_k^\dagger c_k|\mathbf{p},A>}{2\omega_k <\mathbf{p},A|\mathbf{p},A>} \delta(k_0-\omega_k) \tag{34}$$

in terms of which the mesonic energy reads

$$p_0^{\rm mes} = \int \frac{d^4k}{(2\pi)^4} k_0 S_\sigma^0(k, \mathbf{p}) \quad . \tag{35}$$

The previous result (29) may then be generalized by considering the total 4-momentum, whose expression

$$p_\mu = \int \frac{d^4k}{(2\pi)^4} k_\mu \left\{ S(k, \mathbf{p}) + S_\sigma^0(k, \mathbf{p}) \right\} \tag{36}$$

immediately follows from the above definitions. In (36) S and S_σ^0 can clearly be interpreted as the 4-momentum distributions for nucleons and mesons respectively.

Note the asymmetric nature of (36): $S(k, \mathbf{p})$, as defined in eq. (30), embodies the exact spectrum of the system, whereas $S_\sigma^0(k, \mathbf{p})$ contains the free meson spectrum. Thus $S(k, \mathbf{p})$ provides all the needed informations about the interacting system: what is left out is the hamiltonian of the free mesons. Of course one could have started by defining an exact spectral function for the mesons, which would then contain the full spectrum of the system: then a free spectral function for the nucleons would be left out.

An energy integration leads to the 3-momentum distribution of the fermions

$$n(\mathbf{k}, \mathbf{p}) = \sum_l |\phi_l(\mathbf{k}, \mathbf{p})|^2 \tag{37}$$

and of the mesons

$$n_\sigma(\mathbf{k}, \mathbf{p}) = \frac{< \mathbf{p}, A | c_k^\dagger c_k | \mathbf{p}, A >}{2\omega_k < \mathbf{p}, A | \mathbf{p}, A >} \tag{38}$$

respectively, so that the total momentum $\mathbf{p}$ of the nucleus will be

$$\mathbf{p} = \int \frac{d^3k}{(2\pi)^3} \mathbf{k} \left\{ n(\mathbf{k}, \mathbf{p}) + n_\sigma(\mathbf{k}, \mathbf{p}) \right\} \quad . \tag{39}$$

Finally, note that S, S_σ^0 and $S_{\rm vac}$ are Lorentz-covariant and the normalization of S is

$$\int \frac{d^4k}{(2\pi)^4} S(k, \mathbf{p}) = \int \frac{d^3k}{(2\pi)^3} n(\mathbf{k}, \mathbf{p}) = A \quad . \tag{40}$$

Clearly

$$\int \frac{d^4k}{(2\pi)^4} S^0_\sigma = \int \frac{d^3k}{(2\pi)^3} n_\sigma(\mathbf{k}, \mathbf{p}) = N_\sigma \tag{41}$$

provides the average number of mesons inside the nucleus.

3 A Simple Case

In the previous section we have outlined a Lorentz-invariant scheme for treating the atomic nucleus. In practical cases however one is often forced to resort to a single-particle approximation (Dirac-Hartree-Fock approximation[12, 17]). In so doing translational invariance is generally lost and the mesonic degrees of freedom are dropped out of the problem.

Here we examine, in a simple case, how a description of the nucleus solely in terms of nucleonic degrees of freedom can be worked out respecting Lorentz and translational invariance.

For this purpose we start by writing the purely nucleonic state vector [18] (sometimes also called Fock's line)

$$|\mathbf{p}, A>_F = (2\pi)^3 \sqrt{\frac{2p_0}{A!}} \tag{42}$$
$$\int \prod_{i=1}^{A} \frac{d^3k_i}{(2\pi)^3} \frac{m}{\epsilon_{k_i}} \psi_{\boldsymbol{p}}(k_1, \ldots, k_A) a^\dagger_{k_1} \ldots a^\dagger_{k_A} |0> \delta(\mathbf{p} - \sum_{i=1}^{A} \mathbf{k}_i)$$

where $\mathbf{k_i} \equiv (\mathbf{k}_i, s_i)$, s_i embodies the spin-isospin variables, $a^\dagger_{k_i}$ and a_{k_i} are the nucleon creation and annihilation operators, $\epsilon_k = \sqrt{\mathbf{k}^2 + m^2}$ and the vacuum definition

$$a_{ks}|0>= 0 \qquad\qquad b_{ks}|0>= 0 \tag{43}$$

is assumed to hold.

With the normalization (12) for the states $|\mathbf{p}, A>$, and owing to (11), the normalization of (42) reads

$$(2\pi)^3 \int \prod_{i=1}^{A} \frac{d^3k_i}{(2\pi)^3} \frac{m}{\epsilon_{k_i}} |\psi_{\boldsymbol{p}}(k_1, \ldots, k_A)|^2 \, \delta(\mathbf{p} - \sum_{i=1}^{A} \mathbf{k}_i) = 1 \tag{44}$$

where $\psi_{\boldsymbol{p}}(\mathbf{k})$ is also fully antisymmetric in its arguments.

The advantage of (42) is of rendering straightforward the connection with the non relativistic theory: indeed to recover the latter it is sufficient to replace $\psi_{\boldsymbol{p}}(k_1,\ldots,k_A)$ with, e.g., a shell model wave function in momentum space.

In addition (42) allows us to express the energy of the nucleus and the 4-momentum distribution of the nucleons in terms of the 3-momentum distribution only. In fact since the 3-momentum distribution reads

$$n_s^F(\mathbf{k},\mathbf{p}) = \frac{m}{\epsilon_k}\frac{{}_F<\mathbf{p},A|a^\dagger_{ks}a_{ks}|\mathbf{p},A>_F}{{}_F<\mathbf{p},A|\mathbf{p},A>_F} \tag{45}$$

(s being the spin index), the energy will be given by

$$\begin{aligned} p_0^F &= \frac{{}_F<\mathbf{p},A|\int\frac{d^3k}{(2\pi)^3}\frac{m}{\epsilon_k}\epsilon_k a^\dagger_{ks}a_{ks}|\mathbf{p},A>_F}{{}_F<\mathbf{p},A|\mathbf{p},A>_F} \\ &= \sum_s\int\frac{d^3k}{(2\pi)^3}\epsilon_k n_s^F(\mathbf{k},\mathbf{p})\ . \end{aligned} \tag{46}$$

Note that the Lorentz-invariant 3-momentum distribution (45) is normalized according to

$$\sum_s\int\frac{d^3k}{(2\pi)^3}\frac{m}{\epsilon_k}n_s^F(\mathbf{k},\mathbf{p}) = A\ , \tag{47}$$

as it easily follows from (44), and that the 4-momentum distribution is expressed in terms of the latter as follows

$$S^F(k,\mathbf{p}) = 2\pi\sum_s n_s^F(\mathbf{k},\mathbf{p})\delta(k_0-\epsilon_k) + 8\pi\Omega\delta(k_0+\epsilon_k) \tag{48}$$

(of course $S^0_\sigma = 0$).

Also, for sake of completeness, we write down the expressions for n_s^F and p_0^F in terms of the function $\psi_p(k)$:

$$n_s^F(\mathbf{k},\mathbf{p}) = (2\pi)^3 A\int\prod_{i=2}^{A}\frac{d^3k_i}{(2\pi)^3}\frac{m}{\epsilon_k}\left|\psi_p(k_s,k_2,\ldots,k_A)\right|^2\delta(\mathbf{p}-\mathbf{k}-\sum_{j=2}^{A}\mathbf{k}_j) \tag{49}$$

and

$$p_0^F = (2\pi)^3\int\prod_{i=1}^{A}\frac{d^3k_i}{(2\pi)^3}\frac{m}{\epsilon_k}\left|\psi_p(k_1,\ldots,k_A)\right|^2\epsilon_{k_1}\delta(\mathbf{p}-\sum_{j=1}^{A}\mathbf{k}_j)\ . \tag{50}$$

The problem with (42) is that it can never correspond to a bound state. Indeed, in the rest frame, the energy of a bound system clearly satisfies the condition

$$\epsilon < mA \quad . \tag{51}$$

On the other hand, from (46) and (47) it follows

$$p_0^F = \sum_s \int \frac{d^3k}{(2\pi)^3} \epsilon_k n_s^F(\mathbf{k}, \mathbf{p}) > m \sum_s \int \frac{d^3k}{(2\pi)^3} n_s^F(\mathbf{k}, \mathbf{p}) = mA \, . \tag{52}$$

The above result is obvious because the mesons do not intervene when the expectation value of the hamiltonian is taken in the state (42). However it points to the need of inserting the mesons in the model, preserving, possibly, a structure like (42) for the nuclear state. To illustrate how this scope can be achieved in a simple scheme, let us consider an infinite Fermi gas at zero temperature in the rest frame.

Notice that the well known momentum distribution for such a system, namely

$$n_s^F(\mathbf{k}) = \Omega\theta(k_F - k) \quad , \tag{53}$$

is not Lorentz-covariant. Yet we are willing to sacrifice temporarily Lorentz covariance in order to take advantage of the transparency of the Fermi gas model.

We start then from the equations of motion [19, 20]:

$$\begin{aligned} \left(\Box - m_\sigma^2\right) \sigma_{\rm cl} &= g < \overline{\psi}\psi > & (54) \\ (i\gamma \cdot \partial - m - g\sigma_{\rm cl})\, \psi &= 0 \quad . & (55) \end{aligned}$$

These, for a uniform system (the Fermi gas), simplify to

$$\begin{aligned} \sigma_{\rm cl} &= -\frac{g}{m_\sigma^2} < \overline{\psi}\psi > \;, & (56) \\ (i\gamma \cdot \partial - \tilde{m})\, \psi &= 0\,, & (57) \\ \tilde{m} &= m - \frac{g^2}{m_\sigma^2} < \overline{\psi}\psi > \;. & (58) \end{aligned}$$

Thus an effective mass, always lower than the bare one no matter which sign the coupling constant has, naturally appears.

We then perform the canonical transformation

$$\sigma \to \sigma - \sigma_{\rm cl} \tag{59}$$

to get rid of the average value $\sigma_{\rm cl}$: this amounts to replace the free nucleons and antinucleons with dressed ones. These may be viewed as baryons carrying a cloud of σ-mesons, but otherwise unaffected by the interaction according to eq. (57). To evaluate the energy of the system we accordingly use the field representation

$$\psi(\mathbf{y}) = \sum_s \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{m}}{\tilde{\epsilon}_k} \left(\tilde{u}_{ks} \alpha_{ks} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} + \tilde{v}_{ks} \beta^\dagger_{ks} e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \right) \tag{60}$$

where the operators α and β create and annihilate dressed nucleons and antinucleons ($\tilde{\epsilon}_k = \sqrt{\mathbf{k}^2 + \tilde{m}^2}$). Proceeding now along the previously outlined scheme, but utilizing the representation (60), which entails a vacuum different from (43) and, consequently, a different subtraction, we get

$$\begin{aligned} p_0 &= \frac{1}{< \mathbf{p}, A | \mathbf{p}, A >} \sum_s \int \frac{d^3k}{(2\pi)^3} \tilde{\epsilon}_k \frac{\tilde{m}}{\tilde{\epsilon}_k} < \mathbf{p}, A | \alpha^\dagger_{ks} \alpha_{ks} + \beta_{ks} \beta^\dagger_{ks} | \mathbf{p}, A > \\ &+ \frac{1}{2} \Omega m_\sigma^2 \sigma_{\rm cl}^2 - 4\Omega \int \frac{d^3k}{(2\pi)^3} \tilde{\epsilon}_k \ . \end{aligned} \tag{61}$$

Here the first term has the lower bound $\tilde{m}A < mA$, whereas the second one arises from the classical meson field via the canonical transformation (59). Thus a bound nuclear state becomes possible, depending upon the value of the coupling constant g and of the nuclear density. The last term in (61) is again due to the lack of a suitably defined normal product and its structure reflects the presence of two vacua, one associated with the free particles and the other with the dressed ones.

4 General case

In this section we shall examine how a many-body theory for a finite system can be cast in a relativistic framework still preserving Poincaré invariance.

4.1 The Single-particle Green's Function

We start from the single-particle Green's function

$$G_{\boldsymbol{p}}(y,y') = -i\frac{< \mathbf{p}, A|T\left\{\psi(y), \psi^\dagger(y')\right\}|\mathbf{p}, A >}{< \mathbf{p}, A|\mathbf{p}, A >} \tag{62}$$

which obeys the Dyson's equation

$$(i\gamma\cdot\partial - m)\, G_{\boldsymbol{p}}(y,y') + \int d^4z \Sigma_{\boldsymbol{p}}(y,z) G_{\boldsymbol{p}}(z,y') = \gamma_0 \delta^4(y-y') \quad . \tag{63}$$

Since we employ momentum eigenstates $G_{\boldsymbol{p}}$ can only depend upon $y'-y$ (translational invariance). It will carry however an index $\mathbf{p}$ representing the total momentum of the nucleus (the same applies to the self-energy $\Sigma_{\boldsymbol{p}}$ as well). Accordingly, we can work in the 4-momentum space, where eq. (63) becomes

$$(\gamma\cdot k - m - \Sigma_{\boldsymbol{p}}(k))\, G_{\boldsymbol{p}}(k) = \gamma_0 \quad . \tag{64}$$

Obviously, eqs. (63) and (64) are meaningful only if a prescription is given for calculating the self-energy. This is far from being trivial in a relativistic context. A perturbative approach will not work, since a bound state of a finite assembly of nucleons cannot be obtained from a non interacting system via the summation of Feynman diagrams. Rather a collection of nucleons bound in an external (possibly self-consistent) well (as in refs. [12, 17]) might be taken as a starting point. This however entails to give up Lorentz and translational invariance from the very beginning in the hope that the summation of appropriate classes of perturbative diagrams will restore those symmetries. If this turns out to be the case, then eqs. (63) and (64) are meaningful.

Here our standpoint is that $\Sigma_{\boldsymbol{p}}$ exists, is calculable, Lorentz-covariant and transforms like a γ-matrix. As a consequence $G_{\boldsymbol{p}}$ will exist as well. However to write down a Dyson's equation of the type

$$G_{\boldsymbol{p}} = G^0_{\boldsymbol{p}} + G^0_{\boldsymbol{p}} \Sigma_{\boldsymbol{p}} G_{\boldsymbol{p}} \, , \tag{65}$$

preserving at the same time the Poincaré invariance together with the existence of a bound nuclear state, turns out to be a task beyond reach.

Therefore, we start from the spectral representation of $G_{\boldsymbol{p}}$ itself, namely

$$G_{\boldsymbol{p}}(k) = \frac{2p_0}{<\mathbf{p},A|\mathbf{p},A>} \left\{ \sum_l \frac{\phi_l(\mathbf{k},\mathbf{p})\phi_l(\mathbf{k},\mathbf{p})^*}{k_0-\varepsilon_l(\mathbf{k},\mathbf{p})-i\eta} + \sum_l \frac{\chi_l(\mathbf{k},\mathbf{p})\chi_l(\mathbf{k},\mathbf{p})^*}{k_0-\tilde{\varepsilon}_l(\mathbf{k},\mathbf{p})+i\eta} \right\} , \tag{66}$$

which introduces, in parallel to the previously discussed $\phi_l(\mathbf{k},\mathbf{p})$, a new function

$$\chi_l(\mathbf{k},\mathbf{p}) = \tilde{C}_l(\mathbf{k},\mathbf{p}) <\mathbf{p}+\mathbf{k},(A+1)_l|\psi^\dagger(0)|\mathbf{p},A> , \tag{67}$$

with

$$\tilde{C}_l(\mathbf{k},\mathbf{p}) = \sqrt{\frac{M_l^{A+1}}{2p_0\epsilon_l^{A+1}(\mathbf{p}+\mathbf{k})}} , \tag{68}$$

whose modulus square yields the probability of creating a nucleon (or destroying an antinucleon) with 4-momentum k_l on the nuclear ground state $|\mathbf{p},A>$ leaving the residual system in the quantum state $|\mathbf{p}+\mathbf{k},(A+1)_l>$. Moreover, $\epsilon_l^{A+1}(\mathbf{p}+\mathbf{k})$ and $\tilde{\varepsilon}_l(\mathbf{k},\mathbf{p})$ refer to an excited state with baryonic number $A+1$ and are defined in analogy with the $A-1$ case.

It helps, in grasping the significance of the singularities in the propagator, to think to the infinite Fermi gas where one has

$$\begin{aligned} G^F_{\boldsymbol{p}=0}(k) = \frac{/k+m}{2\sqrt{k^2+m^2}} \Bigg\{ & \frac{\theta(k-k_F)}{k_0-\sqrt{k^2+m^2}+i\eta} \\ & + \frac{\theta(k_F-k)}{k_0-\sqrt{k^2+m^2}-i\eta} - \frac{1}{k_0+\sqrt{k^2+m^2}-i\eta} \Bigg\} . \end{aligned} \tag{69}$$

Here the antinucleon states are entirely embodied in the last term of the rhs. This separation however becomes impossible when the interaction is introduced. Accordingly in (66) the antinucleons are actually embodied in both terms of the rhs (not only in the first).

To better understand the above spectral representation it might help to think to a system in which either a nucleon has been annihilated or an antinucleon has been created. This picture strictly speaking is not correct, since the nucleus is actually described by a superposition of states with different number of nucleons and antinucleons; we can however start by considering an assembly of noninteracting nucleons in the rest frame and then annihilate a nucleon or create an antinucleon. In this way two sets of states are obtained, denoted, with self-explaining notation, $(A-1N)$ and $(A+1\tilde{N})$ respectively.

Then switching on the interaction we generate two classes of physical states, still somewhat linked to the original $(A-1N)$ and $(A+1\tilde{N})$ ones. The positive energy solutions are associated to the former, the negative energy ones to the latter, both stemming from the singularities of the first term on the rhs of (66). The singularities of the second term relate to $(A+1N)$ states, i.e. states in which a $N\overline{N}$ pair exists.

Of course

$$
\begin{aligned}
-i\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4} G_{\boldsymbol{p}}(k) &= \frac{2p_0}{<\mathbf{p},A|\mathbf{p},A>} \sum_l \int \frac{d^3k}{(2\pi)^3} |\phi_l(\mathbf{k},\mathbf{p})|^2 \\
&= \frac{A}{\Omega} + 4\int \frac{d^3k}{(2\pi)^3} \, . \qquad (70)
\end{aligned}
$$

By inserting then (66) in (64) one obtains the following eigenvalue equation for $\phi_l(\mathbf{k},\mathbf{p})$ in terms of the $\boldsymbol{\alpha},\beta$ representation for the Dirac's matrices:

$$
\begin{aligned}
& G_{\boldsymbol{p}}^{-1}(k)\phi_l(\mathbf{k},\mathbf{p}) = \qquad (71) \\
& = \left(\varepsilon_l(\mathbf{k},\mathbf{p}) - \boldsymbol{\alpha}\cdot\mathbf{k} - \beta m - \beta\Sigma_p(k)\Big|_{k_0=\varepsilon_l(\mathbf{k},\mathbf{p})}\right)\phi_l(\mathbf{k},\mathbf{p}) = 0 \, .
\end{aligned}
$$

An analogous equation holds as well for the $\chi_l(\mathbf{k},\mathbf{p})$.

In (71) the eigenvalue $\varepsilon_l(\mathbf{k},\mathbf{p})$, as told by eq. (25), represents the difference between the ground state energy of the system in the state $|\mathbf{p},A>$ (moving with total momentum $\mathbf{p}$) and the energy of the system in the state $|\mathbf{p}-\mathbf{k},(A-1)_l>$ (moving with momentum $\mathbf{p}-\mathbf{k}$ and intrinsically left in the quantum state l). It can be regarded as the energy of the quasi-hole. On the same footing $\tilde{\varepsilon}_l(\mathbf{k},\mathbf{p})$ represents the energy of a quasi-particle.

4.2 The Self-Energy and the Final State Interaction

The crucial quantity in the many-body equation for the spectral amplitude is clearly the self-energy. In this subsection we shall obtain the latter from the interaction between a nucleon and the daughter nucleus, thus linking the self-energy to the so-called final-state interaction. An attempt in the same direction has been performed in ref. [3].

Let us introduce a new field operator $\Psi(Y)$ associated with the $A-1$ nucleus viewed as a composite fermion which can exist in different internal

states:

$$\Psi(Y)=\sum_l\int\frac{d^3p}{(2\pi)^3}\frac{M_l^{A-1}}{\epsilon_l^{A-1}(\mathbf{p})}U_{l\boldsymbol{p}}(\mathbf{Y})\mathcal{A}_{l\boldsymbol{p}}\ . \tag{72}$$

Here the operator $\mathcal{A}_{l\boldsymbol{p}}$ annihilates a nucleus of baryonic number $A-1$ in the excited state l and with total 3-momentum $\mathbf{p}$ (we neglect, as irrelevant in the energy range of actual interest, the component of the field that creates the corresponding antinucleus). $U_{l\boldsymbol{p}}(\mathbf{Y})$ is a Dirac spinor, normalized in Fourier transforms according to

$$\begin{aligned} U_{l\boldsymbol{p}}(\mathbf{Y}) &= U_{l\boldsymbol{p}}e^{i\boldsymbol{Y}\cdot\boldsymbol{p}} \\ U^\dagger_{l\boldsymbol{p}}U_{l'\boldsymbol{p}} &= \frac{\epsilon_l^{A-1}(\mathbf{p})}{M_l^{A-1}}\delta_{ll'} \quad . \end{aligned} \tag{73}$$

Next we introduce the four-point Green's function

$$\mathcal{G}_{\boldsymbol{p}}(y,Y;y',Y')=\frac{<\mathbf{p},A|T\left\{\psi(y)\Psi(Y)\psi^\dagger(y')\Psi^\dagger(Y')\right\}|\mathbf{p},A>}{<\mathbf{p},A|\mathbf{p},A>} \tag{74}$$

which obeys the following Bethe-Salpeter-like equation

$$\begin{aligned} &\mathcal{G}_{\boldsymbol{p}}(y,Y;y',Y')=\mathcal{G}^0_{\boldsymbol{p}}(y,Y;y',Y')-i\int d^4y_1\,d^4y_2\,d^4Y_1\,d^4Y_2 \\ &\times\mathcal{G}^0_{\boldsymbol{p}}(y,Y;y_1,Y_1)\mathcal{V}_{\boldsymbol{p}}(y_1,Y_1;y_2,Y_2)\mathcal{G}_{\boldsymbol{p}}(y_2,Y_2;y',Y') \end{aligned} \tag{75}$$

where $\mathcal{G}^0_{\boldsymbol{p}}$ is the propagator of the A^{th} nucleon and of the $(A-1)$ daughter nucleus in the absence of their mutual interaction $\mathcal{V}_{\boldsymbol{p}}$.

Since all the functions $\mathcal{G}_{\boldsymbol{p}}$, $\mathcal{G}^0_{\boldsymbol{p}}$ and $\mathcal{V}_{\boldsymbol{p}}$ carry a definite total momentum $\mathbf{p}$, they are best handled in Fourier transforms: accordingly let us set

$$\begin{aligned} \mathcal{F}(k,k';p)=&\int d^4y\,d^4y'\,d^4Y\,d^4Y' \\ &e^{ik\cdot y}e^{-ik'\cdot y'}e^{i(p-k)\cdot Y}e^{-i(p-k')\cdot Y'}\mathcal{F}_{\boldsymbol{p}}(y,Y;y',Y') \end{aligned} \tag{76}$$

where $\mathcal{F}$ can be $\mathcal{G}$, $\mathcal{G}^0$ or $\mathcal{V}$. In this representation $\mathcal{G}^0$ reads

$$\mathcal{G}^0(k,k';p)=(2\pi)^4\delta(k-k')g^0(k)G^0_{A-1}(p-k)\ . \tag{77}$$

Here $g^0(k)$ is the relativistic free nucleon propagator in the vacuum (not to be confused, of course, with the G^0 appearing in eq. (65)). Indeed the latter is ill-defined, whereas the former (disregarding renormalization effects) reads

$$\left[g^0(k)\right]^{-1}=k_0-\boldsymbol{\alpha}\cdot\mathbf{k}-\beta m\ \ . \tag{78}$$

The relativistic free propagator for the $(A-1)$ nucleus (not interacting with the A^{th} nucleon), is given by

$$G^0_{A-1}(p) = \sum_l \frac{M_l^{A-1}}{\epsilon_l^{A-1}(\mathbf{p})} \frac{U_{l\boldsymbol{p}} U^\dagger_{l\boldsymbol{p}}}{p_0 - \epsilon_l^{A-1}(\mathbf{p}) + i\eta} \ . \tag{79}$$

The Fourier transformed Bethe-Salpeter equation reads then

$$\begin{aligned} \mathcal{G}(k,k';p) &= (2\pi)^4 g^0(k) G^0_{A-1}(p-k)\delta(k-k') \\ &+ g^0(k) G^0_{A-1}(p-k) \int d^4k \mathcal{V}(k,k;p)\mathcal{G}(k,k';p) \end{aligned} \tag{80}$$

We introduce now the Bethe-Salpeter wave function in line with the usual convention [21, 22, 23]

$$\varphi_{\boldsymbol{p}}(y,Y) = -i < 0|T\left\{\psi(y)\Psi(Y)\right\}|\mathbf{p}, A >= \int \frac{d^4k}{(2\pi)^4} \varphi_{\boldsymbol{p}}(k) e^{-iky - i(p-k)Y} \tag{81}$$

whose spectral representation reads

$$\begin{aligned} &\varphi_{\boldsymbol{p}}(k) = \\ &(2\pi)^3 < 0|\psi(0) \frac{\delta(\mathbf{k} - \hat{\mathbf{P}})}{k_0 - H + i\eta} \Psi(0) + \Psi(0) \frac{\delta(\mathbf{k} - \mathbf{p} - \hat{\mathbf{P}})}{k_0 - p_0 + H - i\eta} \psi(0)|\mathbf{p}, A > , \end{aligned} \tag{82}$$

$\hat{\mathbf{P}}$ being the 3-momentum operator and $p_0 = \sqrt{M_A^2 + \mathbf{p}^2}$ both in eqs. (81) and (82).

From (80) an equation for $\varphi_{\boldsymbol{p}}(k)$ is then deduced by writing down the Lehmann representation of $\mathcal{G}(k,k';p)$ and selecting those contributions having the vacuum as intermediate state. This scope is achieved by integrating over p_0 around a small circle surrounding the point $p_0 = \sqrt{\mathbf{p}^2 + M_A^2}$ which indeed corresponds to the pole of the propagator having the vacuum as an intermediate state. Thus we find

$$\oint \frac{dp_0}{2\pi i} \mathcal{G}(k,k';p) = \varphi^*_{\boldsymbol{p}}(k)\varphi_{\boldsymbol{p}}(k') \tag{83}$$

and since $\mathcal{G}^0$ is regular at $p_0 = \sqrt{\mathbf{p}^2 + M_A^2}$ one gets, by integrating eq. (80) over p_0,

$$\varphi_{\boldsymbol{p}}(k) = -i \oint \frac{dp_0}{2\pi i} g^0(k) G^0_{A-1}(p-k) \int \frac{d^4k'}{(2\pi)^4} \mathcal{V}(k,k';p)\varphi_{\boldsymbol{p}}(k') \ . \tag{84}$$

Remarkably, the Bethe-Salpeter wave function $\varphi_{\boldsymbol{p}}(k)$ turns out to be connected with the spectral amplitude $\phi_l(\mathbf{k},\mathbf{p})$. In fact, as seen from eq. (82), $\varphi_{\boldsymbol{p}}(k)$ is a meromorphic function in the variable k_0, with poles located at the energies of the free baryons (first term of the rhs of eq. (82)) and at the energies of the quasiparticle (second term). We can thus insert a complete set of eigenstates of the $(A-1)$ system in the second term of the rhs of eq. (82), integrate over a small circle around the selected pole and, by reminding the definition (23) of ϕ_l, we find

$$\oint \frac{dk_0}{2\pi i}\varphi_{\boldsymbol{p}}(k) = \left(\frac{M_L^{A-1}}{\epsilon_l^{A-1}(\mathbf{p}-\mathbf{k})}\right) U_{l\boldsymbol{p}-\boldsymbol{k}} \frac{1}{C_l(\mathbf{k},\mathbf{p})}\phi_l(\mathbf{k},\mathbf{p}) \; . \tag{85}$$

From the above, using (73) and (82), the connection we are searching for, namely

$$\phi_l(\mathbf{k},\mathbf{p}) = C_l(\mathbf{k},\mathbf{p}) \oint \frac{dk_0}{2\pi i} U^\dagger_{l(\boldsymbol{p}-\boldsymbol{k})}\varphi_{\boldsymbol{p}}(k) \tag{86}$$

follows.

Next we act on (84) with $[g^0]^{-1}$ and again select the poles by integrating over k_0 around the l^{th} eigenvalue of the $(A-1)$ system getting

$$\begin{aligned}(\varepsilon_l(\mathbf{k},\mathbf{p}) - \boldsymbol{\alpha}\cdot\mathbf{k} - \beta m)\,\phi_l(\mathbf{k},\mathbf{p}) = \\ C_l(\mathbf{k},\mathbf{p}) \int \frac{d^4k'}{(2\pi)^4} U^\dagger_{l(\boldsymbol{p}-\boldsymbol{k})}\mathcal{V}(k_l,k';p)\varphi_{\boldsymbol{p}}(k') \; .\end{aligned} \tag{87}$$

The above equation provides a tool for calculating microscopically the self-energy: indeed given an effective nucleon-nucleus interaction $\mathcal{V}(k,k';p)$ the Bethe-Salpeter wave function $\varphi_{\boldsymbol{p}}(k)$ can be found and the product of the baryon self-energy with the spectral amplitude $\phi_l(\mathbf{k},\mathbf{p})$ can be determined.

Note also that the finite size of the system is actually embedded in $\mathcal{V}(k,k';p)$.

Clearly for $\mathcal{V}(k,k';p)$ a model is needed. Interestingly, if we choose the expression

$$\mathcal{V}(k,k';p) = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')(\beta\sigma+\omega) \tag{88}$$

for the effective interaction, then the well known self-energy of the QHD, namely

$$\Sigma_{\boldsymbol{p}}(k) = \sigma + \beta\omega \; , \tag{89}$$

is recovered, σ and ω being the standard scalar and vector fields of QHD.

4.3 The Structure of the Self-Energy

Having examined a microscopical model for the self-energy and its link with the final state interaction, we now write down its general expression, namely

$$\Sigma(k^2, k \cdot p) = S + \gamma_\mu V^\mu + \frac{1}{2M_A} [\gamma_\mu, \gamma_\nu] T^{\mu\nu} , \tag{90}$$

which fulfills the constraints imposed by Poincaré covariance. Our purpose is to explore the predictions that can be made on the relativistic quasiparticle spectrum without specifying the details of the baryon self-energy. In (90)

$$V^\mu = k^\mu V + p^\mu V' \tag{91}$$

and

$$T^{\mu\nu} = k^\mu p^\nu T \tag{92}$$

are a Lorentz vector and a second rank tensor respectively, whereas S, V and V' are scalars, in general complex, whose dependence is upon the invariants k^2 and $k \cdot p$. Alternatively, we can choose as independent variables, k^2 and $(p-k)^2$: this might be more convenient as it will be discussed in sect. 4.5.

Our expression for $\Sigma_{\boldsymbol{p}}$ is similar to the one of ref. [16], often considered in the literature, but we remind that translational invariance strongly constrains its structure. We also remind that terms with γ_5 are neglected as parity-violating; concerning the impact of other discrete symmetries on $\Sigma_{\boldsymbol{p}}$ we refer the reader to [16].

With the self-energy (90) the inverse propagator entering eq. (71) reads

$$G_{\boldsymbol{p}}^{-1}(k) = (1-V)\left\{k_0 - vp_0 - \boldsymbol{\alpha}\cdot(\mathbf{k} - v\mathbf{p}) - \beta m^* - \frac{t}{2M_A}\beta\,[\not k, \not p]\right\} \tag{93}$$

where the definitions

$$m^* = \frac{m+S}{1-V} , \tag{94}$$

$$v = \frac{V'}{1-V} , \tag{95}$$

and

$$t = \frac{T}{1-V} \tag{96}$$

have been introduced. The above Green's function $G_{\boldsymbol{p}}$ describes a moving quasiparticle interacting with the daughter nucleus. However, before discussing the physical significance of the functions introduced so far, namely m^*, v, t and V, we first need to find the poles of the Green's function, i.e. to solve the eigenvalue equation for the spectral amplitude ϕ_l, and then to study the analytical properties of $\Sigma_{\boldsymbol{p}}$.

4.4 Solution of the Eigenvalue equation

To solve eq. (71) we look for the poles of the propagator

$$G_{\boldsymbol{p}}(k) = \frac{1}{(1-V)\left(k^2 - m_1^2 + i\eta\right)} \left\{k_0 - vp_0 - \boldsymbol{\alpha}\cdot(\mathbf{k} - v\mathbf{p}) + \beta m^* - \frac{t}{2M_A}\left[\not{k}, \not{p}\right]\beta\right\} \tag{97}$$

where a new mass

$$m_1^2 = m^{*2} + 2vk\cdot p - v^2p^2 + \left(k^2 - \left(\frac{k\cdot p}{M_A}\right)^2\right)t^2 \tag{98}$$

has been introduced. The small positive imaginary part in the denominator is meant to be effective as far as m^*, v and t are real.

Clearly the poles of $G_{\boldsymbol{p}}$ coincides with the eigenvalues $\varepsilon_l(\mathbf{k}, \mathbf{p})$ of (71) and are the roots of the equation

$$k^2 = m_1^2(k^2, k\cdot p) \quad . \tag{99}$$

We shall discuss the regularity of the functions m^*, v and t later on. In the regions where they are well behaved and m_1^2 is positive definite the nonlinear equation (99) might display positive

$$\varepsilon_l^+ = \sqrt{\mathbf{k} + m_1^2}\,|_{k_0=\varepsilon_l^+} \tag{100}$$

and negative

$$\varepsilon_l^- = -\sqrt{\mathbf{k} + m_1^2}\,|_{k_0=\varepsilon_l^-} \tag{101}$$

solutions. Notice that these in the rest frame become

$$\varepsilon_l^{\pm} = M_A v_l \pm \sqrt{(1-t_l^2)\mathbf{k}^2 + m_l^{*2}} \tag{102}$$

where the suffix l reminds that the various quantities have to be evaluated at $k_0 = \varepsilon_l^{\pm}$.

We thus see that v_l yields an overall frequency shift, whereas m_l^* sets the splitting between the two branches of the spectrum. Noteworthy is that an energy shift, a mass renormalization and the introduction of an effective momentum

$$\mathbf{k}_{\text{eff}} = \sqrt{1-t^2}\,\mathbf{k} \tag{103}$$

allow a recovering of the structure of the Dirac's equation eigenvalues. In particular the scalar component of $\Sigma_{\boldsymbol{p}}$ renormalizes the mass, the vector one partly renormalizes the Green's function residua and partly shifts the spectrum and finally the tensor term modifies the momentum.

Of course m_1^2 is not an even function of k_0 (due to its dependence upon $p \cdot k$), so that in general $\varepsilon_l^+ \neq -\varepsilon_l^-$. Also, the existence of one solution does not imply, in general, the existence of the other one.

From the analogy with the Dirac's equation we can interpret these solutions as describing the propagation of a nucleon (or of a hole, depending upon the boundary conditions) or of an antinucleon in the medium.

Finally we note that the positive branch is the meaningful one in the non relativistic limit, where

$$\varepsilon_l(\mathbf{k}) \approx m_l^* + M_A v_l + \frac{(1-t_l^2)\mathbf{k}^2}{2m^*}\ . \tag{104}$$

Now we search for the spinor structure of the spectral amplitude ϕ_l. The equation (71) can be recast, with the help of (97), in the form

$$\left\{\varepsilon_l(\mathbf{k},\mathbf{p}) - v_l p_0 + \boldsymbol{\alpha}(\mathbf{k} - v_l\mathbf{p}) + \beta m_l^* - \frac{t_l}{2M_A}\left[{/\!\!\!k}\,|_{k_0=\varepsilon_l(\mathbf{k},\mathbf{p})}, {/\!\!\!p}\right]\beta\right\}\phi_l(\mathbf{k},\mathbf{p}) = 0 \tag{105}$$

which defines $\phi_l(\mathbf{k},\mathbf{p})$ up to a normalization.

Now for positive frequency solutions we get

$$\phi_l^+(\mathbf{k},\mathbf{p}) = \sum_s \left(1 + \frac{\gamma\cdot p}{M_A}t\right) u_s \mathcal{N}_{ls}^+(\mathbf{k},\mathbf{p})\,\Big|_{k_0=\varepsilon_l^+} \tag{106}$$

where

$$u_s = \sqrt{\frac{\mathcal{E}+\mathcal{M}}{2\mathcal{M}}}\begin{pmatrix}\varphi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}}{\mathcal{E}+\mathcal{M}}\varphi_s\end{pmatrix}\ , \tag{107}$$

$$\boldsymbol{\mathcal{K}} = \mathbf{k}(1-t^2) - \mathbf{p}\left(v - \frac{m^*}{M_A}t - \frac{k\cdot p}{M_A^2}t\right), \tag{108}$$

$$\mathcal{M} = m^* - \frac{t}{M_A}\left(k\cdot p - vp^2\right) \tag{109}$$

and

$$\mathcal{E} = \sqrt{\mathcal{K}^2 + \mathcal{M}^2}\,. \tag{110}$$

For negative frequency solutions we have instead

$$\phi_l^-(\mathbf{k},\mathbf{p}) = \sum_s \left(1 + \frac{\gamma\cdot p}{M_A}t\right) v_s \mathcal{N}_{ls}^-(\mathbf{k},\mathbf{p})\,\Big|_{k_0=\varepsilon_l^-} \tag{111}$$

with

$$v_s = \sqrt{\frac{\mathcal{E}+\mathcal{M}}{2\mathcal{M}}}\begin{pmatrix} -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{K}}}{\mathcal{E}+\mathcal{M}}\varphi_s \\ \varphi_s \end{pmatrix}\,. \tag{112}$$

In (107) and (112) φ_s is the Pauli spinor corresponding to the spin polarization s. We normalize the spinors u and v according to $\overline{u}u = 1$, $\overline{v}v = -1$, while the normalization coefficient $\mathcal{N}$ remains to be fixed. This is done by observing that near the pole $\varepsilon_l^{(\pm)}(\mathbf{k},\mathbf{p})$ the Green's function behaves like

$$G_{\boldsymbol{p}} \simeq \frac{2p_0}{<\mathbf{p}A|\mathbf{p}A>}\frac{\phi_l^\pm(\mathbf{k},\mathbf{p})\phi_l^{\pm\dagger}(\mathbf{k},\mathbf{p})}{k_0 - \varepsilon(\mathbf{k},\mathbf{p}) - i\eta}\,, \tag{113}$$

but on the other hand

$$G_{\boldsymbol{p}} = \left\{k_0 - \boldsymbol{\alpha}\cdot\mathbf{k} - \beta m - \beta\Sigma\right\}^{-1} \quad . \tag{114}$$

By comparing (113) and (114) near the pole the relation

$$\frac{2p_0}{<\mathbf{p}A|\mathbf{p}A>}\left|\phi_l^\pm(\mathbf{k},\mathbf{p})\right|^2 = \frac{1-\frac{vp_0}{k_0}}{(1-V)\left(1-\frac{m_1}{k_0}\frac{\partial m_1}{\partial k_0}\right)}\Bigg|_{k_0=\varepsilon_l^\pm}\,, \tag{115}$$

which fixes the normalization of $\phi_l^\pm$, follows.

4.5 Properties of the Self-Energy

In this subsection we first shortly study the analytic properties of the self-energy starting from its microscopic definition. For this purpose we consider the equation of motion for the field operator $\psi(y)$

$$(i\gamma\cdot\partial - m - \mathcal{U})\,\psi(y) = 0 \tag{116}$$

where the operator $\mathcal{U}$ is defined through the equal time commutator

$$
\left(\mathcal{U}\psi\right)(y) = [\psi(y), H'(y_0)] \ . \tag{117}
$$

The operator $\mathcal{U}$ here introduced is model dependent. For our hamiltonian (1) it reads

$$
(\mathcal{U}\psi)(y) = g\sigma(y)\psi(y) \ . \tag{118}
$$

Now, by operating with the operator on the lhs of (116) on the Green's function definition and by exploiting the Dyson's equation (63), one obtains

$$
\begin{aligned}
\left(\Sigma G\right)(y, y') &\equiv \int d^4z \Sigma(y,z)G(z,y') \qquad (119)\\
&= 2ip_0 \frac{< A\mathbf{p}|T\left\{\left(\mathcal{U}\psi\right)(y)\psi^\dagger(y')\right\}|A\mathbf{p}>}{<\mathbf{p},A|\mathbf{p},A>} \ .
\end{aligned}
$$

From (119) the Lehmann representation

$$
\begin{aligned}
\Sigma(k)G(k) = &\frac{2M_A}{<A|A>}\times \qquad (120)\\
&\left\{\sum_l \frac{<A|\psi^\dagger(0)|-\mathbf{k},(A-1)_l><-\mathbf{k},(A-1)_l|(\mathcal{U}\psi)(0)|A>}{k_0-\varepsilon_l(\mathbf{k})-i\eta}\right.\\
&\left.+\sum_l \frac{<A|(\mathcal{U}\psi)(0)|-\mathbf{k},(A+1)_l><-\mathbf{k},(A-1)_l|\psi^\dagger(0)|A>}{k_0-\tilde{\varepsilon}(\mathbf{k})+i\eta}\right\}
\end{aligned}
$$

is deduced *in the rest frame.*

Let us now restrict ourselves to the discrete spectrum only. Here, as a function of k_0, $G(k)$ (which, strictly speaking, is a distribution) displays a pole in correspondence of each discrete eigenvalue. Owing to (120) so does ΣG, hence Σ must be regular. Furthermore a zero of G certainly exists between two neighboring poles, where instead ΣG is regular. Hence Σ *must diverge where G vanishes.* On the other hand in the continuous spectrum region G is finite, nonvanishing and complex on the real axis: as a consequence the same occurs for Σ.

In conclusion Σ, and consequently m^*, v, t and V, are well-behaved functions in the neighborhoods of any point of the spectrum of G. However they are singular elsewhere on the real axis (actually between the poles of G), in correspondence to the discrete part of the spectrum of the nucleus, whose occurrence is allowed by its finite size.

Now let us shortly comment the dependence of the eigenvalues upon $\mathbf{k}$ and $\mathbf{p}$, which is fixed by eq. (25) (at least for the advanced part of the spectrum; the generalization however is obvious):

$$\varepsilon_l(\mathbf{k},\mathbf{p}) = \sqrt{\mathbf{p}^2 + M_A^2} - \sqrt{(\mathbf{p}-\mathbf{k})^2 + (M_l^{A-1})^2} \ . \tag{121}$$

This relation suggests the choice of k^2 and $(p-k)^2$ as independent variables for expressing m^*, v, t and V, because it entails

$$(p-k)^2 = \left(M_l^{A-1}\right)^2 \ . \tag{122}$$

Now the eigenvalues satisfy eq. (99), which, setting $f = k^2 - m_1^2$, can be recast as follows:

$$f((p-k)^2, k^2) = 0 \tag{123}$$

which in turn can be inverted to yield

$$(p-k)^2 = \varphi_l(k^2) \ . \tag{124}$$

Combining then (122) with (124) one finally gets

$$\varphi_l(k^2) = \left(M_l^{A-1}\right)^2 \quad \forall k^2 \tag{125}$$

which constrains the functional dependence of the self-energy.

As a simple example fulfilling eq. (124) we consider the case $v = 1$ and $t = 0$ (which implies the absence of an effective momentum) and requiring in addition to have m^* depending upon $(p-k)^2$ only. Then the eigenvalue equation (99) reduces to

$$(p-k)^2 = m^{*2}((p-k)^2) \ . \tag{126}$$

In this simple example the only unconstrained quantity left out is V, which can then be fixed by comparing with the experimental data.

In conclusion the purpose of this subsection has been to point out how the structure of the self-energy is constrained, in a Lorentz covariant traslationally invariant framework for a finite system, by the analytic properties it must display and by its dependence upon the momentum p of the nucleus.

5 Conclusions

In this paper we propose a relativistic, hamiltonian theory of nuclei, which also fulfills translational invariance.

In order to set up such a framework we focused our attention on the spectral amplitude: indeed by one side the latter represents a most suitable tool for extending a basic concept of the nuclear physics in the non-relativistic domain, namely the one of single particle orbital, to the relativistic regime, on the other side it turns out to be natural to express the energy of the nucleus and its 3- and 4-momentum distributions, in a scheme respecting covariance and in which the number of nucleons is not conserved, precisely in terms of the spectral amplitude.

We achieve our scope within a model, namely assuming the nucleus to be a composite system of nucleons and mesons. Although the latter are not elementary degrees of freedom, yet we believe that they can be considered as "effective" building blocks useful in describing the nuclear structure in a quite wide kinematical region: therefore to them we have first applied our formalism. On the other hand if and when more fundamental degrees of freedom, namely the quarks and gluons, should be accounted for, then our theory can equally well be applied to this new dynamical situation.

As an aside, in sect. 3 of the present paper we have also discussed the limiting case, opposite to the above one, where the energies involved are so small that the nucleus lends itself to be described solely in terms of (suitably renormalized) nucleonic degrees of freedom. This is the extreme situation corresponding to the more traditional nuclear physics where the mesonic degrees of freedom are eliminated in favour of static forces acting between the nucleons. Of course the limits of such a scheme are met when the Meson Exchange Currents (MEC) intervene in a decisive manner, as, e.g., in the electrodisintegration of the deuteron at large momentum transfer [24, 25].

In the present paper we have not addressed the question related to the global gauge invariance of the theory, a symmetry required to consistently account for both the currents and the forces carried by the mesons. We have rather centered our attention on the dynamics of the many-body system exploring in particular the fermionic self-energy, the basic quantity entering into the definition of the fermion propagator. More specifically we started from its relativistic, translational invariant definition and by combining its Lehmann representation with the relativistic Dirac's equation we succeeded

in deriving an equation obeyed by the spectral amplitude ϕ_l itself. This one can be solved once the self-energy is known and the associated eigenvalues directly provide the energy spectrum of the nucleus. We have actually done that assuming a general Poincaré invariant self-energy: thus we obtained the spectral amplitude and the spectrum of the nucleus. Notably the latter turned out to be a shifted Dirac's spectrum with an effective mass and momentum.

In attempting a more microscopic approach, in particular to account for the final state interaction, we considered a Bethe-Salpeter-type equation whose solution happens to be closely connected with the spectral amplitude ϕ_l. Worth noticing is that our Bethe-Salpeter approach, for a particular choice of the nucleon-nucleus interaction and for infinite nuclear matter, reduces to QHD.

Finally we have explored the analytical properties of the self-energy in the energy variable. We have found that, in correspondence of the zeros of the fermion propagator, the self-energy displays simple poles, which are clearly connected with the bound states of the discrete part of the nuclear spectrum. The locations of the poles is dependent upon the momentum $\mathbf{p}$ of the nucleus, which is perfectly defined in our translational invariant framework. The possibility is thus offered of exploring how the spectrum of the nucleus modifies as a function of $\mathbf{p}$: this seems to us a fascinating topic to investigate, in particular we conjecture that in the infinite momentum frame the nuclear spectrum will considerably simplify.

In conclusion in the present paper we have outlined a theoretical framework suitable for handling relativistic finite nuclear system. The next step would be to deal with concrete applications: for example we can use our scheme to ascertain the antinucleonic components in the nuclear wave function, which has never been properly assessed. Furthermore, as already mentioned, there is no reason preventing the extension of the present formalism to describe a system made by quarks and gluons, because the constrains we obtain on the self-energy only stem from Lorentz and translational invariance. In fact the structure of a quark self-energy in a nucleon should be simpler, because the excitation spectrum of the nucleon is simpler than the one of a nucleus.

Clearly the deep inelastic scattering on both the nucleon and the nucleus offer the best testing ground of the present formalism, which might provide a help in achieving a better understanding of those processes.

A Appendix: Boosting the spectral amplitude

We remind the definition

$$\phi_l(\mathbf{k}, \mathbf{p}) =< (A-1)_l, \mathbf{k}-\mathbf{p}|\psi(0)|A, \mathbf{p} > \quad . \tag{127}$$

Let Λ be a pure boost which transforms the state $|A, \mathbf{p}>$ to a state at rest, i.e.

$$\Lambda|A, \mathbf{p}>= |A, \mathbf{p}=0> \quad . \tag{128}$$

Obviously

$$\phi_l(\mathbf{k}, \mathbf{p}) =< (A-1)_l, \mathbf{k}-\mathbf{p}|\Lambda^{-1}\left(\Lambda\psi(0)\Lambda^{-1}\right)\Lambda|A, \mathbf{p}> \quad . \tag{129}$$

We also know that

$$\Lambda\psi(0)\Lambda^{-1} = S\psi(0) \quad . \tag{130}$$

where

$$S = e^{(-i/4)\sigma_{\alpha\beta}\omega^{\alpha\beta}} \tag{131}$$

and

$$\sigma_{\alpha\beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta] \quad . \tag{132}$$

the parameters of a pure boost having the form $\omega^{\alpha\beta} \to \omega^{0i}$.

The matrix S transforming a particle at rest (with mass m) into one moving with momentum $\mathbf{q}$ is

$$S(\mathbf{q}) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} 1 & \frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{E+m} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{E+m} & 1 \end{pmatrix} \tag{133}$$

with $E = \sqrt{q^2+m^2}$. The velocity of the moving particle is

$$v = \frac{q}{\sqrt{q^2+m^2}} = \frac{q}{E} \tag{134}$$

In our case the boost is characterized by the velocity

$$\mathbf{v} = \frac{\mathbf{p}}{\sqrt{p^2+M_A^2}} \tag{135}$$

It is now immediate to conclude that

$$\Lambda\psi(0)\Lambda^{-1} = S(\frac{m}{M_A}\mathbf{p})\psi(0) \tag{136}$$

Finally we need $\Lambda|(A-1)_l, \mathbf{p}-\mathbf{k}>$. It clearly transforms in another state having the same internal quantum numbers but a transformed momentum, namely

$$\mathbf{p}-\mathbf{k} \rightarrow \mathbf{q} = \frac{\sqrt{p^2+M_A^2}}{M_A}(\mathbf{p}-\mathbf{k}) - \frac{\sqrt{(\mathbf{p}-\mathbf{k})^2-(M_{A-1}^l)^2}}{M_A}\mathbf{p} \tag{137}$$

We define in term of $\mathbf{q}$ the quantity

$$\tilde{\phi}(\mathbf{q}) =< (A-1)_l, \mathbf{q}|\psi(0)|A, 0> \tag{138}$$

and the previously derived transformation properties provide us the factorization

$$\phi_l(\mathbf{k},\mathbf{p}) = S\left(\frac{m}{M_A}\mathbf{p}\right)\tilde{\phi}(\mathbf{q}) \tag{139}$$

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